Math 70 3.4 & 3.5 Rates of Change & Slope

3.4 Rates

Objectives

1) Determine rates of change
2) Use units analysis

3.5 Slope of a Line

Objectives

1) Interpret a slope as a rate, with units
2) Use the slope formula to determine the slope of a line
3) Find slopes of vertical & horizontal lines

Graphing calculator

1) Avoiding round-off error
   GC-7

2) Optional: Value, Trace, Zoom Integer
   GC-18

Units Analysis

Objective: Use units to check answers and formulas

Units describe what type of items or measurements are used. Ex: feet, minutes, liters, revolutions, miles per hour, dollars per square foot, cubic feet per minute.

<u>Rules for units or dimensional analysis</u>

1. To add or subtract units, the terms being combined must have the same units, and the result has those same units. (This is the same as "combining like terms".)

   Example: feet + feet = feet

   Example: 2 apples + 3 apples = 5 apples

2. When multiplying units, the dimension of the units increases. The exponent on the units is called the "dimension".

   Example: feet x feet = square feet = $feet^2$

3. The word "per" in the units means divide.

   Example: miles per hour = $\frac{miles}{hour}$ (the rate a car moves)

   Example: dollars per square foot = $\frac{dollars}{feet^2}$ (the cost of carpeting)

   Example: cubic feet per minute = $\frac{feet^3}{\text{min}}$ (rate a bathtub fills with water)

4. If you multiply a "per" expression by another expression with the same units as the denominator, the units cancel. (This is like canceling or dividing out a common factor.)

   Example: $\frac{miles}{hour} \cdot hours = miles$ (because 'hour' and 'hours' are the same, they cancel out)

   Example: $\frac{\$2}{foot^2} \cdot 17\,feet^2 = \$34$ (because square feet cancel out)

5. To check an equation or formula
   a. The simplified units must be the same on the left side as on the right side.
   b. Substitute the units for each quantity into the equation.
   c. Constants and constant coefficients have no units.

   Example: $D = R \cdot T$, where distance D is given in miles, rate R is given in miles per hour, and time is given in hours, means the dimensions in the equation are: $miles = \frac{miles}{hour} \cdot hours$, the hours cancel, and we have miles = miles.

Example: Suppose we mis-remembered the formula as $D \cdot R = T$. Dimensional analysis gives $miles \cdot \frac{miles}{hour} = hours$, but the left side simplifies to $\frac{miles^2}{hour} \neq hours$, helping us realize that we have the formula wrong, because the units on the left are not the same as the units on the right.

6. Exponents or dimensions tell us the shape of the concept being calculated
   a. A one-dimensional calculation is length of a line or a curved line, measured in one-dimensional units, like feet or meters.

      Example: the distance from point A to point B

      Example: the length of one side of a rectangle

      Example: the distance around the edge of a circle, the circumference $C = 2\pi r$, where C and r are measured in meters and 2 and π are constant coefficients having no units, has dimensional analysis $meters = (no\,units)(no\,units) \cdot meters$

      Example: the cost of fencing $\frac{\$2}{meter}$ tells us we want length or perimeter.

   b. A two-dimensional calculation is area, the flat space contained in a flat object, measured in two-dimensional units, like square feet= $feet^2$

      Example: the area of a rectangle $A = L \cdot W$ has dimensional analysis $feet^2 = feet \cdot feet$

      Example: the cost per square foot of carpet $\frac{\$2}{foot^2}$ tells us that we need the area of the space to be carpeted

   c. A three-dimensional calculation is volume, the voluminous space contained within a real-world object, measured in three-dimensional units, like cubic meters = $meters^3$

      Example: the volume of a rectangular solid $V = L \cdot W \cdot H$ has dimensional analysis $feet^3 = feet \cdot feet \cdot feet$

<u>Practice:</u>
Use dimensional analysis to determine if the following formulas and calculations might be correct.

1) The perimeter of a rectangle is $P = 2L + 2W$

2) The surface area of a rectangular solid is $A = 2LW + 2WH + 2LH$

3) The volume of a sphere is $V = \frac{4}{3}\pi r^3$

4) The total cost of fencing = cost per foot times the number of square feet.

These topics are all review from Math 45.

① Find the slope of a line passing through $(0, -\frac{1}{3})$ and $(-2, \frac{5}{6})$.

Step 1: Write the slope formula $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Step 2: Choose which point to call (x_1, y_1)

$(0, -\frac{1}{3}) \qquad (-2, \frac{5}{6})$

$(x_1, y_1) \qquad (x_2, y_2)$

CAUTION: Be consistent, using the same point (pair) for x_1 as used for y_1.

↳ If your slope has the wrong sign, you may have used subscript 2 where you should have used subscript 1, or vice-versa.

Step 3: Substitute the four values and simplify

$$m = \frac{\frac{5}{6} - \left(-\frac{1}{3}\right)}{-2 - 0}$$

$$= \frac{\left(\frac{7}{6}\right)}{-2}$$

$$= \frac{7}{6} \div -2$$

$$= \frac{7}{6} \cdot \frac{-1}{2}$$

fractions can be done on GC.

$$\boxed{m = \frac{-7}{12}}$$

(2) Find the slope of the line $3x - 4y = 7$.

Goal: $y = mx + b$ form

Method: isolate y.

$$3x - 4y = 7$$
$$-3x \qquad\qquad -3x$$
$$\frac{-4y}{-4} = \frac{-3x}{-4} + \frac{7}{-4}$$
$$y = \boxed{\frac{3}{4}}x - \frac{7}{4}$$

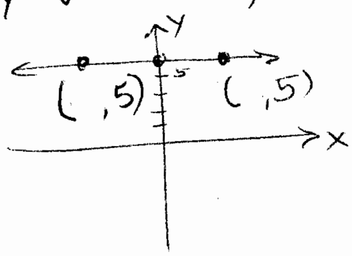
↑ m is the x-coefficient

$$\boxed{m = \frac{3}{4}}$$

(3) Find the slope of the line $y = 5$

(skip)

y-variable, no x-variable ⇒ horizontal



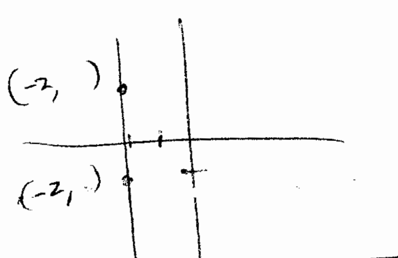
$$m = \frac{5-5}{\#} = \frac{0}{\#} = 0$$ for all horizontal lines

$$\boxed{m = 0}$$

(4) Find the slope of the line $x = -2$

(skip)

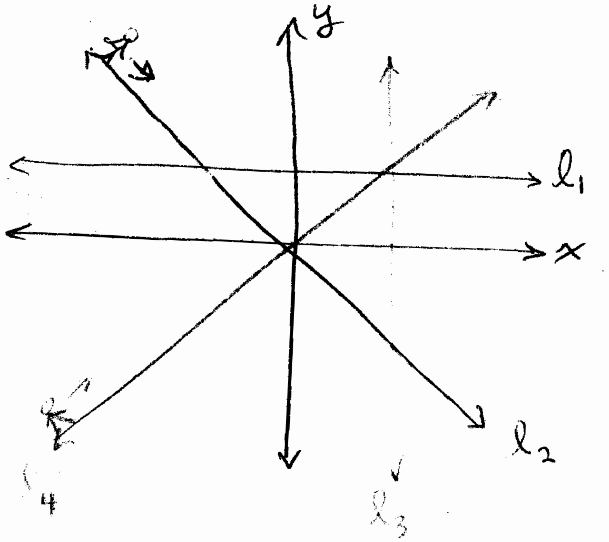
x-variable, no y-variable ⇒ vertical



$$m = \frac{\#}{-2-(-2)} = \frac{\#}{0} =$$ undefined for all vertical lines

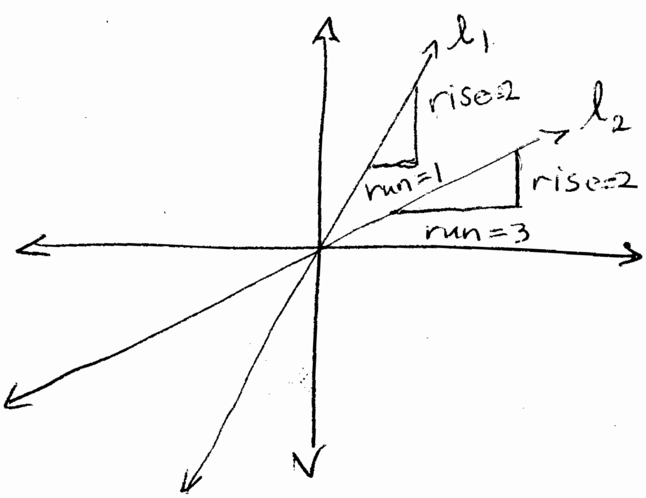
$$\boxed{m = \text{undefined}}$$

(5) Given these four lines, match each to the correct description of its slope.



slope	
positive	l_4 uphill L→R
negative	l_2 downhill L→R
zero	l_1 horizontal
undefined	l_3 vertical

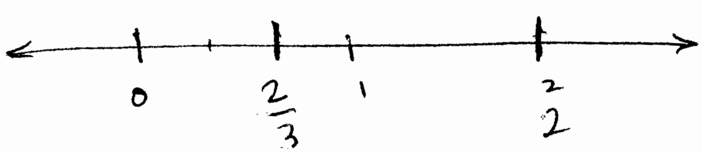
(6) Which line has greater slope?



$m_1 = \frac{2}{1} = 2$

$m_2 = \frac{2}{3}$

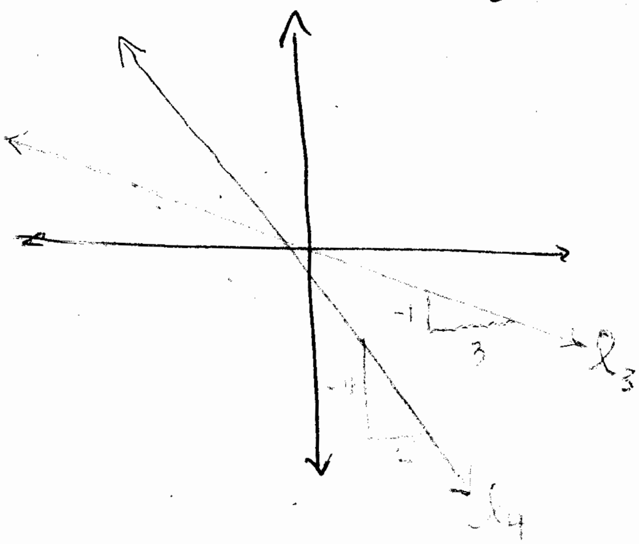
Want to find numerical values of slope for each line, put or imagine those numbers on a number line, and choose the one which is greater — more to the right.



⇑

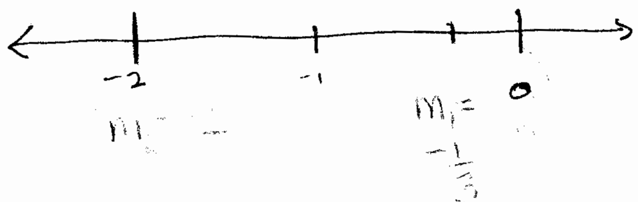
l_1 has the greater slope.

(7) Which line has greater slope?



$m_3 = \frac{-1}{3}$

$m_4 = \frac{-4}{2} = -2$

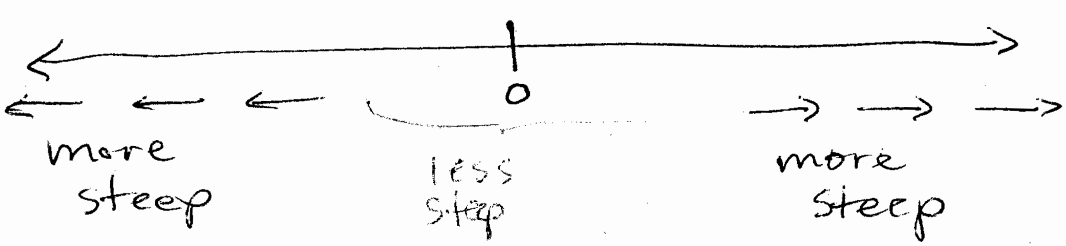


The number to the right is the greatest $\rightarrow -\frac{1}{3}$

means $\boxed{\ell_3 \text{ has greater slope.}}$

Considering "steepness" $\rightarrow$ when both lines have positive slopes, the steeper line has greater slope. But when both lines have negative slopes, the steeper line has lesser slope.

Slopes near zero are less steep.
The further we go away from zero, the more steep the line.



M170

(8) The price of one adult 1-day pass to Disney World is given by $f(x) = 2.7x + 38.64$, where x is the number of years since 1996.

a) Predict the ticket price in 2015.

b) Interpret the slope.

c) Interpret the y-int.

a) Find value of x by subtracting

$$\begin{array}{r} 2015 \\ -1996 \\ \hline 19 \end{array}$$

Evaluate $f(19) = 2.7(19) + 38.64$

$= \boxed{\$89.94}$ Answer

b) $\text{Slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x}$ ← units of $f(x) = y$ / ← units of x

slope $= \dfrac{\$2.70}{\text{yr}}$

Slope positive ⇒ quantity y is increasing
Slope negative ⇒ quantity y is decreasing

2.7 is positive so this slope means.

Answer: The price of a ticket increases (1) $ (2) 2.70 (3) each year (4).

(1) increase / decrease
(2) units y
(3) slope
(4) units x

} 4 components of correct answer.

c) Y int → value of y when x = 0.

Answer: y-int $38.64 is price of ticket in 1996

(9)

The number of people y (in thousands) employed as nurses in the US can be estimated by $-266x + 10y = 27409$
where x is the number of years after 2000.
a) Find the slope and y int.
b) What does the slope mean? Interpret
c) What does the y-int mean? Interpret

The equation is written in standard form $(ax + by = c)$ instead of slope-intercept form $(y = mx + b)$, so rearrange.

$-266x + 10y = 27409$ — isolate y (Solve for y)

$+266x$ $\qquad$ $+266x$

(266x and 27409 are not like terms.)

$$10y = 266x + 27409$$

$$\frac{10y}{10} = \frac{266x}{10} + \frac{27409}{10}$$

$$y = \underbrace{26.6}_{\text{slope}}x + \underbrace{2740.9}_{\text{y-int}}$$

a) slope = 26.6
y-int = 2740.9 or (0, 2740.9) y-int

b) Interpret ⇒ include units first.
x ⇒ # of years after 2000, so x=0 means 2000.
y ⇒ # people employed as nurses, in thousands

y-intercept (0 years after 2000, 2740.9 thousand people)

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y-int means: In 2000, 2740.9 thousand people were employed as nurses.

Also valid:

2740.9 thousand people = (2740.9)(1000) people
= 2,740,900 people

In 2000, 2,740,900 people were employed as nurses.

Slope: $m = \frac{\Delta y}{\Delta x} = \frac{\text{units of } y}{\text{units of } x}$ = units of y per units of x

m = 26.6 thousand people employed as nurses per year (since 2000)

Slope is the rate of change of y with respect to x

{So if $m > 0$ is positive, the quantity y is increasing

{and if $m < 0$ is negative, the quantity y is decreasing

m means "The number of people employed as nurses is increasing by 26.6 thousand (or 26,600) people per year.

Name ______________________________

Date ______________________________

TI-84+ GC 7 Avoiding Round-off Error in Multiple Calculations

Objectives: Recall the meaning of exact and approximate
Observe round-off error and learn to avoid it
Perform calculations using the order of operations and extra parentheses

Recall: An exact answer has no error. If we use an exact result to perform additional calculations, we'll continue to get exactly the right answer. If we perform the same calculation to different versions of an exact answer, we'll always get the same, exact final result.

An approximate answer is close to the exact answer, but is a "near miss". We usually find approximate answers by rounding or approximating. If we start with an approximate answer and perform additional calculations, we'll get approximate final results.

CAUTION: You should always give an EXACT answer unless the instructions tell you to round.

Round-off error is the absolute value of the difference between the exact answer and a rounded approximation of that answer, given by this formula: $RoundoffError = |exact - approximate|$.
Round-off error is the answer to the question "How wrong is the rounded answer?"

Example 1: $\frac{1}{8}$ = 0.125 exactly. Rounded to the nearest tenth, $\frac{1}{8} \approx .1$
The round-off error, using the formula, in the answer 0.1 is $|0.125 - 0.1| = 0.025$
In this example, the answer is wrong by 0.025.

Round-off errors can become much bigger if a calculation is done from rounded partial results.

Example 2: To illustrate the error of rounding partial results, calculate $\frac{2472.7908}{0.4678}$ exactly and with rounded partial results, and then find the resulting round-off error. How wrong will the answer be?

a) Calculate $\frac{2472.7908}{0.4678}$ exactly. Answer: 5286

b) Round 2472.7908 to the nearest tenth. Answer: 2472.8

c) Round 0.4678 to the nearest tenth. Answer: 0.5

d) Divide your rounded answer for 2472.7908 by your rounded answer for 0.4678

$$\frac{2472.8}{.5} = 4945.6$$

Answer: 4945.6

e) Find the round-off error for this calculation.

$$|5286 - 4945.6| = 340.4$$

Answer: 340.4

How wrong is the answer? It's off by 340.4! That's a lot.

> IMPORTANT: Do not round parts of a calculation! Instead, do the calculation all at once (using extra parentheses for the correct order of operations) or use memory storage locations.
> If the instructions say to round, round only the final answer.

Example 3:

Calculate $\frac{41.78-3(6.913)^2}{29.188+76.342}$. Round to the nearest thousandth.

> Remember that the long fraction bar means that the entire numerator and entire denominator must be calculated before the results are divided. But the GC follows the order of operations and will not add or subtract before dividing unless we use extra parentheses, like this:
>
> $$\frac{\left(41.78-3(6.913)^2\right)}{(29.188+76.342)}$$

[( ] [4] [1] [.] [7] [8] [−] [3] [×] [6] [.] [9] [1] [3] [^] [2] [ )] [÷] [( ] [2] [9] [.] [1] [8] [8] [+] [7] [6] [.] [3] [4] [2] [ )] [ENTER]

```
(41.78-3*6.913^2
)/(29.188+76.342
)
       -.9626523927
```

Then round the final answer to the nearest thousandth. Answer: -.963

Example 4: Calculate $\frac{(-12)^{3-6}}{9.7-18.034}$. Round to the nearest ten-thousandth.

> Notice that there is a subtraction in the exponent. The GC will not subtract before an exponent unless extra parentheses are added, like this:
>
> $$\frac{(-12)^{(3-6)}}{9.7-18.034}$$
>
> Also notice the long fraction bar, as before. Again, extra parentheses are needed so the GC will subtract the denominator before dividing, like this:
>
> $$\frac{(-12)^{(3-6)}}{(9.7-18.034)}$$

Example 4, continued:

((-) 1 2) ^ (3 − 6) ÷ (9 . 7 − 1 8 . 0 3 4) ENTER

```
(-12)^(3-6)/(9.7
-18.034)
  6.943888933E-5
```

Convert scientific notation to standard notation:
Multiply 6.943888933 by 10^{-5}, by moving the decimal point five places left to get
0.00006943888933
Then round to the nearest ten thousandth (four decimal places):
0.0001

Answer: 0.0001

Example 5: Calculate $50000\left(1+\frac{0.073}{365}\right)^{12(365)}$. Round to the nearest hundredth.

Notice that there is a multiplication in the exponent. The GC will not multiply before an exponent unless extra parentheses are used:

$$50000\left(1+\frac{0.073}{365}\right)^{(12(365))}$$

5 0 0 0 0 (1 + . 0 7 3 ÷ 3 6 5) ^ (1 2 × 3 6 5) ENTER

```
50000(1+.073/365
)^(12*365)
     120053.2527
```

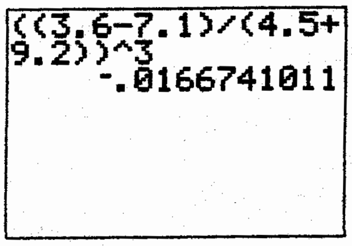
Round to nearest hundredth (2 decimal places): 120,053.25

Answer: 120,053.25

Example 6: Calculate $\left(\dfrac{3.6-7.1}{4.5+9.2}\right)^3$. Round to the nearest thousandth.

> Notice that the parentheses supplied surround both the numerator and the denominator. These do not ensure that the numerator will be subtracted first! To get the correct answer, use additional parenthesis *inside* the given parentheses, like this:
>
> $\left(\dfrac{(3.6-7.1)}{(4.5+9.2)}\right)^3$

((3 . 6 − 7 . 1) ÷ (
4 . 5 + 9 . 2)) ^ 3 ENTER

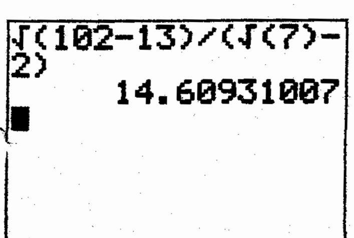


Answer: <u>-0.017</u>

Example 7: Calculate $\dfrac{\sqrt{102-13}}{\sqrt{7}-2}$. Round to the nearest hundredth.

> Notice the square roots are different. First, the square root in the numerator is the square root of a difference – use parentheses around the difference. Second, the denominator is a difference of a square root and 2, so close the parentheses for the square root, and use another set around the entire denominator, like this:
>
> $\dfrac{\sqrt{(102-13)}}{\left(\sqrt{(7)}-2\right)}$

2nd x^2 1 0 2 − 1 3) ÷ (2nd
x^2 7) − 2) ENTER



Answer: <u>14.61</u>

Name ______________________________

Date ______________________________

GC 8 Units Analysis

Objectives: Use units analysis to organize word problems

Units analysis means checking an equation or expression using algebra on the units.
In an equation, the simplified units on the left-hand side of the equation should be the same units as those on the right-hand side of the equation.

A unit which follows the word "per" should be written in the denominator when using units analysis.

Example 1: 32 miles per hour = $\boxed{\frac{32\,miles}{1\,hour}}$

Just as $\frac{23}{23}=1$, the same units in the numerator and denominator divide out.
In units analysis, it doesn't matter if the same units are singular (mile) or plural (miles).
Some instructors use the word "cancel"; other instructors dislike using the word "cancel".

Example 2: $\frac{2\,miles}{1\,mile} =$ $\boxed{2 \text{ (no units)}}$

Units can be multiplied together.

Example 3: The formula D=RT describes the relationship between distance, rate, and time. Calculate the distance covered by a person walking at a rate of 5 miles per hour for time 3 hours and simplify the units using units analysis.

Substitute $R=\frac{5\,miles}{1\,hour}$ and $T=3\,hours$ into the formula: D=$\frac{5\,miles}{1\,hour}\cdot 3\,hours$.

Collect the numbers together and units together: D=$5\cdot 3\frac{hours}{1\,hour}\cdot miles$

Multiply the numbers and divide out the units: D= 15 miles. Answer: D= 15 miles

Example 4: Use the formula D=RT and units analysis to calculate the rate a car is going if it travels 160 miles in 2.5 hours.

Substitute D=160 miles and T=2.5 hours into the formula: $160\,miles = R\cdot 2.5\,hours$

Isolate R by dividing both sides by 2.5 hours: $\frac{160\,miles}{2.5\,hours}=R$

Divide the numbers and change the units to standard English: $64\,\frac{miles}{hour}=R$

Answer: R= 64 mph

Example 5: How many miles can a car that gets 23 mpg (miles per gallon) go on 3 gallons of gas?

Multiply 3 gallons times 23 miles per gallon.

Write the units so the numerator and denominator are clear: $3\,gallons \cdot \frac{23\,miles}{1\,gallon}$

Collect the numbers together and units together: $3 \cdot 23\,miles \cdot \frac{gallons}{1\,gallon}$

Multiply the numbers and divide out the units: $69\,miles$ Answer: 69 miles

> When analyzing units, the simplified units on the left-hand side of the equation must be equal to the simplified units on the right-hand side of the equation.
> This rule can be used to check that formulas are written correctly.

Example 6: When trying to write a formula for gasoline use, a student wrote: $gallons = \frac{miles}{gallon} \cdot miles$

Simplify the units and determine if the student has set up the problem correctly.

$gallons = \frac{miles}{gallon} \cdot miles$ gives $gallons = \frac{miles^2}{gallon}$. The units on the left-hand side are different from the units on the right-hand side.

Answer: $\boxed{gallons = \frac{miles^2}{gallon}}$ The set-up is incorrect.

> Recall: When dividing fractions, multiply by the reciprocal of the second fraction.

Example 7: Calculate $\frac{3}{4} \div \frac{6}{5}$

Multiply by reciprocal $\frac{3}{4} \cdot \frac{5}{6} = \frac{1}{4} \cdot \frac{5}{2} = \frac{5}{8}$ Answer: $\boxed{\frac{5}{8}}$

> When dividing by terms which have units which form a fraction, multiply by the reciprocal.

Example 8: Calculate number of gallons of gasoline needed to travel 420 miles in a car that gets 12 mpg.

Find the formula by arranging the given units so that they make sense:

$miles = \frac{miles}{gallon} \cdot gallons$ gives miles = miles after dividing out the gallons.

Substitute the values for each known quantity and a variable for the unknown quantity:

$$420\,miles = \frac{12\,miles}{gallon} \cdot x\,gallons$$

Example 8, continued :

Isolate the variable by dividing both sides by 12 mpg: $420\, miles \div \frac{12\, miles}{gallon} = x\, gallons$

Divide is multiply by the reciprocal: $420\, miles \cdot \frac{gallon}{12\, miles} = x\, gallons$

Collect the numbers and units: $\frac{420}{12}\, gallon \cdot \frac{miles}{miles} = x\, gallons$

Divide the numbers and units: $35\, gallons = x\, gallons$

(Check that the units on the left side are the same as the units on the right side.)

Answer: 35 gallons

When adding terms that have the same units, treat the units as though it were like terms.

Example 9: Simplify by combining like terms: $2xy + 5xy$

$2xy + 5xy = (2+5)xy = 7xy$

Answer: $7xy$

Example 10: Simplify by combining like terms: 70 gallons minus 32 gallons

$70\, gallons - 32\, gallons = (70-32) \cdot gallons = 38\, gallons$.

Answer: 38 gallons

The units in equations with several operations must obey the order of operations.

Example 11: The gas tank of a car that gets 25 mpg (city) holds 13.8 gallons. If the driver began with a full tank, how many gallons remain after 180 miles of city driving?

Find the formula by arranging the given units so that they make sense:

a) (Gallons in tank) – (gallons used) = (gallons remaining)

Need an expression for gallons used: For example, $miles \cdot \frac{gallons}{mile}$ = gallons.

But the problem gives $\frac{miles}{gallon}$, not $\frac{gallons}{mile}$. We need the reciprocal of the mileage.

b) Divide by mileage (mpg): gallons used = $miles \div \frac{miles}{gallon} = miles \cdot \frac{gallons}{mile}$

Substitute b) into a):

Gallons in tank $-\ miles \div \frac{miles}{gallon}$ = gallons remaining

Substitute: 13.8 gallons – 180 miles $\div \frac{25\, miles}{gallon}$ = gallons remaining

Take reciprocal, collect units and use the order of operations: 13.8 gallons - $\frac{180\, miles}{25\ miles} \cdot gallon$ =

13.8 gallons – 7.2 gallons =

Answer: 6.6 gallons

Practice

Translate and simplify each statement using units.

1) h hours times 50 miles per hour.

2) 37 miles – m miles

3) 13 miles + x hours times 60 miles per hour

4) It's 500 miles to the Grand Canyon. We've been driving 3 hours at 70 mph. How many miles remain?

5) It took us 6 hours to drive to our lodging at Yosemite. After driving 4.5 hours, we went 45 miles per hour. How many miles did we go at 45 mph?

6) 48 miles divided by 24 miles per gallon.

7) 14 gallons minus x miles divided by 24 miles per gallon.

8) A car has a 14-gallon gas tank and averages 24 mpg in the city and 30 mpg on highways.

   (a) If driven 48 miles in the city (no highway driving), how much gasoline will be used?

   (b) If driven x miles in the city, how much gasoline will be used?

   (c) Write an expression for the amount of gas left in the tank in terms of miles driven (x) for city driving (no highway), starting with a full tank.

   (d) How much gas is left in the tank after the first 300 miles of city driving?

   (e) Write an equation and solve: How many miles can be driven until the tank is empty if only driving in the city?

9) A car has a 14-gallon gas tank and averages 24 miles per gallon (mpg) in the city and 30 mpg on highways.

   (a) If he drives x miles on the highway, how much gasoline will be used?

   (b) If he starts with a full tank, write an expression for the amount of gas left in the tank in terms of miles driven (x) for highway driving.

   (c) How much gas is left in the tank after the first 300 miles of highway driving?

   (d) Write an equation and solve: How many miles can he drive before his tank is empty if he is only driving on a highway? [Hint: Which variable is 0 when his tank is empty?]

Solutions

1) $h\,hours \cdot 50\frac{miles}{hour} = 50h \cdot \frac{hours}{hour} \cdot miles$
$= 50h\,miles$

2) (37-m) miles

3) $13\,miles + x\,hours \cdot \frac{60\,miles}{hour}$
$= 13\,miles - 60x\,miles \cdot \frac{hours}{hour}$
$= 13\,miles - 60x\,miles = (13 - 60x)\,miles$

4) $500\,miles - 3\,hours \cdot \frac{70\,miles}{hour}$
$= 500\,miles - 3 \cdot 70\,miles \cdot \frac{hours}{hour}$
$= 500\,miles - 210\,miles = 290\,miles$

5) $6\,hours - 4.5\,hours = 1.5\,hours$
$1.5\,hours \cdot \frac{45\,miles}{hour} = 1.5 \cdot 45\,miles \frac{hours}{hour}$
$= 67.5\,miles$

6) $48\,miles \div 24\frac{miles}{gallon} = 48\,miles \cdot \frac{gallon}{24\,miles}$
$= \frac{48\,miles}{24\,miles}gallons = 2\,gallons$

7) $14\,gallons - x\,miles \div \left(24\frac{miles}{gallon}\right)$
$= 14\,gallons - x\,miles \cdot \frac{gallon}{24\,miles}$
$= 14\,gallons - \frac{x}{24}gallon\frac{miles}{miles}$
$= 14\,gallons - \frac{x}{24}gallon = \left(14 - \frac{x}{24}\right)gallons$

8)

(a) $48\,miles \div \frac{24\,miles}{gallon} = \frac{48}{24} \cdot miles \cdot \frac{gallon}{miles}$
$= 2 \cdot gallons$

(b) $x\,miles \div \frac{24\,miles}{gallon} = \frac{x}{24}gallons$

(c) $14 - \frac{x}{24}$ gallons

(d) $14 - \frac{300}{24} = 1.5$ gallons

(e) $0 = 14 - \frac{x}{24}$; x = 336 miles

9)

(a) $x\,miles \div \frac{30\,miles}{gallon} = \frac{x}{30}miles\frac{gallon}{miles}$
$= \frac{x}{30}gallons$

(b) $14 - \frac{x}{30}$ gallons

(c) $14 - \frac{300}{30} = 4$ gallons

(d) $0 = 14 - \frac{x}{30}$; x = 420 miles

Rev 8-7-13

Name ______________________________

Date ______________________________

TI-84+ GC 18 Cursor, Trace, Value, and Zoom Integer

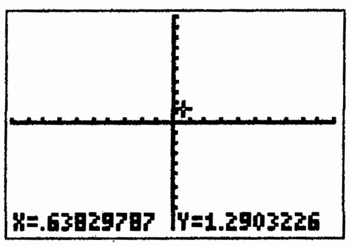
Objectives: Identify the cursor and pixels on a GC graph
Identify the cursor on a GC graph in TRACE mode
Learn the uses and limitations of the TRACE mode
Calculate value at a single point using 'Value'
Calculate values at integers using Zoom Integer (ZInteger)

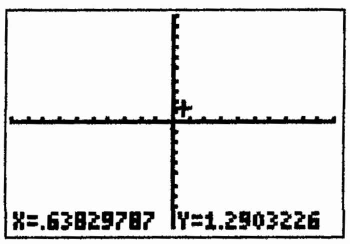
Each rectangular lighted point in a computer or GC screen is called a <u>pixel</u>. The GC screen has about 9025 pixels in a grid. You can move a cursor anywhere in this grid.

Example 1: Move the cursor in an empty graphing window to see x- and y-coordinates of pixels.

Clear your Y= menu and go to a standard graphing window. 

Press  several times and then  several times.

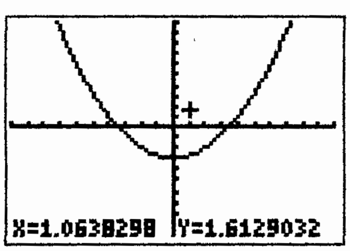




Notice: 1. The cursor moves one pixel each time. 2. The GC shows a cursor shaped like a plus sign where the center point blinks on and off. 3. At the bottom of the screen, the GC gives the x- and y-coordinates of the cursor's location.

Example 2: Graph $y = \frac{1}{4}x^2 - 3$ in a standard window and move the cursor.

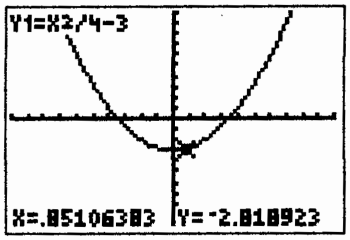
Press  several times and then  several times.
Notice that the cursor behaves exactly as it did in Example 1.

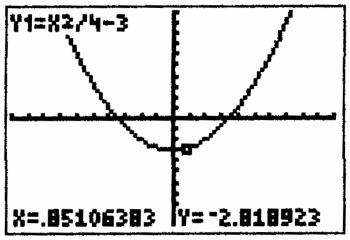


"Trace" is the graphing calculator equivalent of running your finger along the graph and naming the coordinates (x,y) of the point where your finger is.

Example 3: Graph $y=\frac{1}{4}x^2-3$ in a standard window and move the cursor in TRACE mode.

Press [TRACE]. Press [▶] several times, [◀] several times, and [▲] several times.



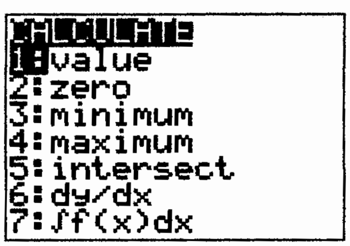


Notice that in trace mode:

1. The cursor moves on the graph, so each point shown satisfies the equation or function.
2. The cursor appears on your graph, but is now a square that flashes between a solid square with legs on each corner to an empty square and back again.
3. [▶] moves to the right one pixel, in the positive x direction, so the x-coordinate increases.
4. [◀] moves to the left one pixel, in the negative x direction, so the x-coordinate decreases.
5. The x-coordinates of pixels often have messy decimals.
6. The y-coordinate changes according to the function or equation; this could be up or down or flat.
7. Pressing [▲] or [▼] does nothing when there's only one equation graphed.

CAUTION: The coordinates found by the TRACE mode are pixel locations. The TRACE mode doesn't find x-intercepts or other useful points – in fact, TRACE often skips over x-intercepts!

The Calculate menu, abbreviated CALC, is a 2nd function above the TRACE button and has seven calculations the GC can do from a graph. Press [2nd] [TRACE] to see this menu:



The first five calculations are useful in algebra; calculations 6 and 7 are for calculus.

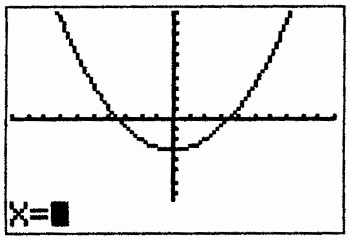
To select value and calculate the function at a chosen value of x, press [1] or [ENTER].

Example 4: Use CALC - value to find y for $y=\frac{1}{4}x^2-3$, when $x=-2.5$

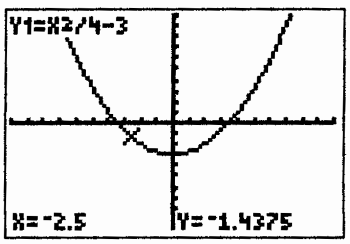
Step 1: Input the desired function in the Y= menu.

Step 2: Press [2nd] [TRACE] to enter the CALC menu.

Step 3: Press [1] or [ENTER] to begin the value calculation.



Step 4: The calculator is waiting for an x-value. Type the desired value: [(−)] [2] [.] [5] [ENTER] The resulting y-coordinate is the desired function value at that value of x.

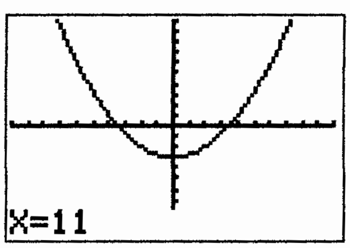


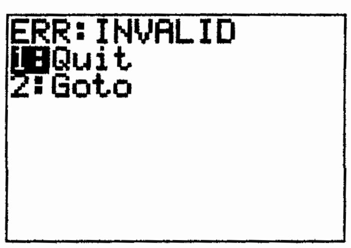
Answer: f(-2.5)=-1.4375

CAUTION: If the x-value is not within the GC window, the value calculation will fail.

Example 5: Try to find y when x=11, in the standard window for $y=\frac{1}{4}x^2-3$ using the Value calculation and observe the error message that results.

CALC menu: [2nd] [TRACE] Value: [ENTER] x-coordinate: [1] [1] [ENTER]

Answer: 



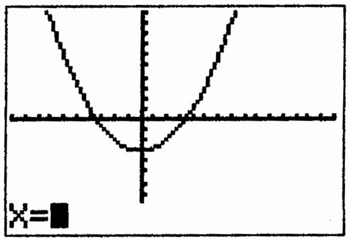
11 is greater than Xmax = 10 in the standard window, so the value calculation failed.

Press [1] to select "Quit" in the Error message menu. See the next example for a successful calculation.

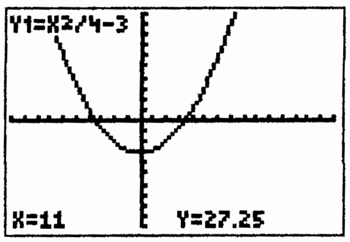
Example 6: Change Xmax to be greater than 11, and find y when x=11 for $y=\frac{1}{4}x^2-3$.

Press [WINDOW]. Change Xmax [▼] [1] [5]. Return to CALC menu [2nd] [TRACE].

Select value [ENTER]. (Notice: the GC returns to the graph automatically.)



Enter the x-coordinate: [1] [1] [ENTER]



Answer: <u>y=27.25</u>

Notice that the y-coordinate, y=27.25, is not visible on the screen (it's greater than Ymax=10), but the value calculation was successful anyway.

To return to the calculating screen, press clear twice [CLEAR] [CLEAR] or quit [2nd] [MODE].

Recall: The <u>integers</u>, sometimes abbreviated Z or J, are the set of numbers with no decimal part: $\{\ldots-3,-2,-1,0,1,2,3\ldots\}$. We often use these values of x when graphing a function by hand. ZOOM INTEGER (ZInteger) makes each pixel equal to 1 unit. This distorts the appearance of the graph, but makes TRACE go from one integer to the next. Notice: The x-values in ZInteger are integers, but the y-values are whatever the function makes them, integers or decimals.

Example 7: Use ZInteger to complete this table for $y=\frac{1}{4}x^2-3$

x	y =f(x) value
-2	
-1	
0	
1	
2	

Example 7, continued:

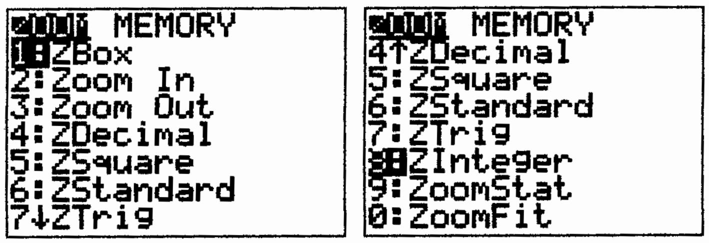
<u>Step 1:</u> Input the function in the Y= menu.

<u>Step 2:</u> Enter the ZOOM menu and select zoom integer.

Press [ZOOM], then press [▼] 7 times to go down to 8.

A quicker way: go over the top of the menu: [▲] [▲] [▲] [ENTER]

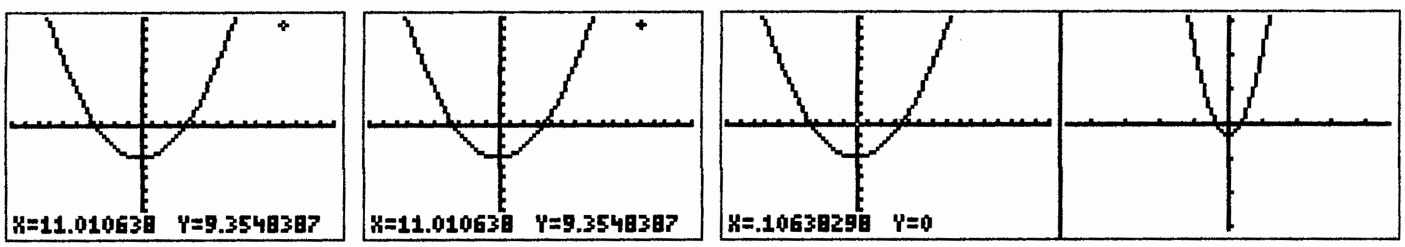
An even quicker way (if you remember ZInteger is 8): just press 8: [8]



<u>Step 3:</u> Use the zoom cursor to select the center of the graph and press enter.

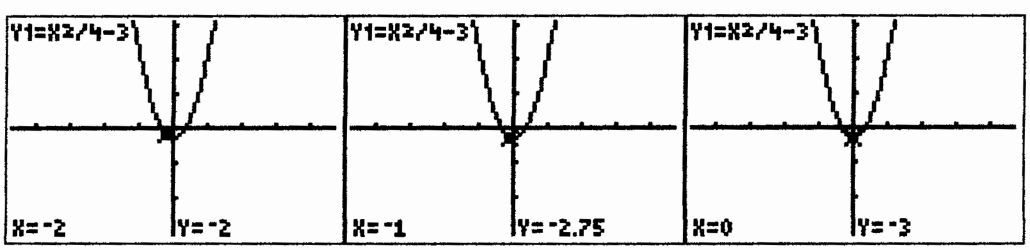
The zoom cursor is a blinking plus symbol, but smaller than the graph cursor.

Move the cursor (by pressing [▲] [▶] [▼] [◀]) to the new center of the graph – in this case, the origin, (0,0). Then press [ENTER].

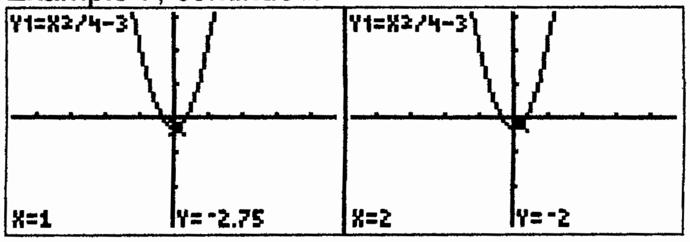


<u>Step 4:</u> Trace in the ZInteger screen.

Press [TRACE], then [▶] and [◀] to move along the curve. Notice that the coordinates of the cursor are displayed, but the x-coordinates are integers. Fill in the y-coordinates in the chart.



Example 7, continued:



Answer:

x	y value
-2	-2
-1	-2.75
0	-3
1	-2.72
2	-2

Summary:

Use Value or ZInteger when you want to see the point on the graph.
Use Value to evaluate any single (or irregularly spaced) decimal values of x.
Use ZInteger when you want to evaluate at many integer values of x.
Use whichever is quicker for you when evaluating at only one or two integer values of x.

Use a table whenever you don't need to see the points on the graph.
Use automatic table for integer or decimal values that are equally spaced.
Use ask table for values that are not equally spaced.

Example 8: Use ZInteger to make a table for $y=\frac{1}{5}x+1$, with integer x-values from -6 to 1.

Input function [Y=] [CLEAR] [X,T,Θ,n] [÷] [5] [+] [1]. Zoom integer [ZOOM] [8]. If necessary, move cursor to the origin, then press [ENTER]. Press [TRACE] and [◄] or [►] to see the needed y-values. (Look closely at decimal points!) Draw and complete table.

Answer:

x	y =f(x) value
-6	-.2
-5	0
-4	.2
-3	.4
-2	.6
-1	.8
0	1
1	1.2

Practice:

1) Explore TRACE mode a bit more:

   a) Graph any function. Press [TRACE], then press [›] 10 or 15 times. What happens to the cursor? What do the coordinates on the screen tell you?

   b) Press [›] and hold it down. What happens to the cursor? What do the coordinates at the bottom of the screen tell you? What (eventually) happens to the window?

   c) Return to the standard window by pressing [ZOOM] [6] again. Are you in TRACE mode now or not?

2) Complete the table for $y = \frac{1}{8}x^2 + 2$.

x	y value
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

3) Complete the table for $y = \frac{1}{8}x^2 + 2$

x	y =f(x) value
-1.5	
-0.5	
0.5	
1.5	
2.5	

4) Complete the table for $y = 0.1x^2 + 0.2$

x	y =f(x) value
-2	
-1.5	
-1	
-0.5	
0	
0.5	
1	
1.5	